Exercise 1

(a) Find y' by implicit differentiation.

(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

$$9x^2 - y^2 = 1$$

Solution

Part (a)

Differentiate both sides with respect to x.

$$\frac{d}{dx}(9x^2 - y^2) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(9x^2) - \frac{d}{dx}(y^2) = 0$$

$$9\frac{d}{dx}(x^2) - \frac{d}{dx}(y^2) = 0$$

$$9(2x) - \left[2y \cdot \frac{d}{dx}(y)\right] = 0$$

$$18x - 2yy' = 0$$

Solve for y'.

$$y' = \frac{9x}{y}$$

Part (b)

Solve for y first.

$$y^2 = 9x^2 - 1$$

$$y = \pm\sqrt{9x^2 - 1}$$

Then take the derivative.

$$\begin{aligned} y' &= \frac{d}{dx}(\pm\sqrt{9x^2 - 1}) \\ &= \pm\frac{1}{2}(9x^2 - 1)^{-1/2} \cdot \frac{d}{dx}(9x^2 - 1) \\ &= \pm\frac{1}{2}(9x^2 - 1)^{-1/2} \cdot (18x) \\ &= \pm\frac{9x}{\sqrt{9x^2 - 1}} \\ &= \frac{9x}{y} \end{aligned}$$